

Solitons in supersymmetry breaking meta-stable vacua

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ABSTRACT: In recently found supersymmetry-breaking meta-stable vacua of the supersymmetric QCD, we examine possible existence of solitons. Homotopy groups of the moduli space of the meta-stable vacua show that there is no nontrivial soliton for $SU(N_c)$ gauge group. When $U(1)_B$ symmetry present in the theory is gauged, we find non-BPS solitonic (vortex) strings whose existence and properties are predicted from brane configurations. We obtain explicit classical solutions which reproduce the predictions. For $SO(N_c)$ gauge group, we find there are solitonic strings for $N \equiv N_f - N_c + 4 = 2$, and \mathbf{Z}_2 strings for the other N . The strings are meta-stable as they live in the meta-stable vacua.

KEYWORDS: Solitons Monopoles and Instantons, D-branes, Supersymmetry Breaking, Supersymmetry and Duality.

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1. Introduction

There are many ways to motivate need of supersymmetries — while one is, needless to say, a solution to the loop-level hierarchy problem, others include a strong controllability of the theories. Renowned Seiberg’s duality [1] among $\mathcal{N} = 1$ supersymmetric gauge theories is a famous example. Furthermore, with the supersymmetries, analysis via string/M theories are accessible from field theories. However, the introduction of the supersymmetries in realistic field theory models always accompanies the problem of how to break them at low energy. In this sense, it should be celebrated that Intriligator, Seiberg and Shih [2] found recently that one of the simplest supersymmetric field theories, the supersymmetric QCD (SQCD), admits supersymmetry-breaking meta-stable vacua via the Seiberg’s duality (some following works include [3–7]). With the help of known realization of the duality in string theories [8] a la Hanany-Witten setup [9] (see [10] for a review), the brane configuration corresponding to the meta-stable vacua have been identified [3–5], even though the vacua break the supersymmetries.

Since this provides a new mechanism to break the supersymmetries, it is important to study classical/quantum properties of this newly found vacua. In particular, the structure of the classical moduli space is directly related to a possible existence of solitons in the vacua. Any soliton, if existent, is quite relevant to particle phenomenology, partially through cosmological evolution and phase transition of the universe.

In this article, we examine the possible existence of solitons, in the supersymmetry-breaking meta-stable vacua. Since the vacua are provided by a Seiberg-dual (called “magnetic” or “macroscopic”) theory of the SQCD, field-theoretical facilitation is concrete enough to determine topology of the moduli space. We find that when the gauge group of the SQCD is $SU(N_c)$, homotopy groups of the moduli space are trivial, thus any soliton does not appear in the vacua.

When the gauge group is $U(N_c)$, mainly because of its diagonal $U(1)$ sector, the vacua admit vortex strings. With a help of the brane realization of the meta-stable vacua [3–5] (see also [6]), the existence of the vortex strings is predicted from brane configurations, as a generalization of Hanany-Tong set-up [11] where vortex strings in $\mathcal{N} = 2$ gauge theories are identified with D2-branes in Hanany-Witten brane configurations. The predicted properties such as tensions, supersymmetries, and species of strings, can be examined by direct construction of vortex string solutions in the magnetic theory of the SQCD. It is noteworthy that the vortex strings are non-BPS (\equiv supersymmetry-breaking). Eventually the magnetic theory is found to be a gauge theory with flavor fields allowing so-called semilocal vortex strings [13], which are our non-BPS vortex strings. Because they are topological, the lifetime of the corresponding strings in the SQCD is of the same order as the meta-stable vacua. When the quark masses of the original SQCD split, various kinds of vortex strings appear with different tensions.

For SO gauge groups, we find similar semilocal strings only when $N_c = N_f + 2$ where the dual gauge group is $SO(2) \sim U(1)$. For generic N_f and N_c we find \mathbf{Z}_2 solitonic strings.

The organization of the paper is as follows. After reviewing the supersymmetry-breaking meta-stable vacua of the magnetic SQCD [2], we examine the homotopy groups of the moduli space of the vacua in section 2. Possible existence of solitons is studied accordingly. Then in section 3, we obtain a brane configuration of the vortex strings in the meta-stable vacua, following the brane realization of the meta-stable vacua [3–5] and also that of the solitonic strings in $\mathcal{N} = 2$ gauge theories [11]. In section 4, we explicitly construct the vortex string solutions in the magnetic SQCD, and study their stability to see the consistency with the brane picture. Some discussions on renormalization groups, relevance to cosmologies, and detailed correspondence to the brane picture are presented in section 5, with a brief conclusion.

2. Solitons in magnetic theory of SQCD

2.1 Meta-stable vacua of SQCD

First, we briefly summarize the main results of Intriligator, Seiberg and Shih [2], the supersymmetry-breaking meta-stable vacua of the magnetic theory of the SQCD.

In [2], $\mathcal{N} = 1$ 4d supersymmetric $SU(N_c)$ Yang-Mills with N_f massive flavors Q and \tilde{Q} (SQCD) was shown to have meta-stable supersymmetry-breaking vacua in the free magnetic range $N_c + 1 \leq N_f \leq \frac{3}{2}N_c$, with sufficiently small quark masses. Along the analysis done in [2], we will work in this range. The dual $SU(N)$ ($N = N_f - N_c$) theory is infra-red free and its low energy effective theory is described by the Kähler potential and the superpotential

$$K = \text{Tr}_c [q e^{-V} q^\dagger + \tilde{q}^\dagger e^V \tilde{q}] + \text{Tr}_f [M^\dagger M], \quad (2.1)$$

$$W = h \text{Tr}_c [q M \tilde{q}] - h \mu^2 \text{Tr}_f M. \quad (2.2)$$

Here M is the meson field and q, \tilde{q} are the dual quarks whose charges are summarized below. (We omit the $U(1)_R$ charge in the table.)

	$SU(N)$	$SU(N_f)_V$	$U(1)_B$
$M_{[N_f \times N_f]}$	1	adj.	0
$q_{[N \times N_f]}$	\square	$\bar{\square}$	1
$\tilde{q}_{[N_f \times N]}$	$\bar{\square}$	\square	-1

The bosonic part of the Lagrangian of the “macroscopic theory” which we call magnetic theory is of the form

$$\mathcal{L} = \text{Tr}_c \left[-\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} - \mathcal{D}_\mu q \mathcal{D}^\mu q^\dagger - \mathcal{D}_\mu \tilde{q}^\dagger \mathcal{D}^\mu \tilde{q} \right] - \text{Tr}_f [\partial_\mu M^\dagger \partial^\mu M] - V \quad (2.3)$$

with a scalar potential $V = V_F + V_D$ given by¹

$$V_F = |h^2| \text{Tr}_f [|\tilde{q}q - \mu^2 \mathbf{1}_{N_f}|^2] + |h^2| \text{Tr}_c [|qM|^2 + |\tilde{q}^\dagger M^\dagger|^2], \quad (2.4)$$

$$V_D = \frac{g^2}{4} \text{Tr}_c \left[(qq^\dagger - \tilde{q}^\dagger \tilde{q})^2 \right] - \frac{g^2}{8} \left(\text{Tr}_c qq^\dagger - \text{Tr}_c \tilde{q}^\dagger \tilde{q} \right)^2. \quad (2.5)$$

Supersymmetric configuration is then given by

$$\tilde{q}q = \mu^2 \mathbf{1}_{N_f}, \quad qM = 0, \quad \tilde{q}^\dagger M^\dagger = 0, \quad qq^\dagger - \tilde{q}^\dagger \tilde{q} = 0. \quad (2.6)$$

The first condition cannot be satisfied when $N < N_f$ because the rank N of the matrix $\tilde{q}q$ is less than N_f (rank condition) [2]. A configuration minimizing the potential is of the form

$$M = \begin{pmatrix} 0_{N \times N} & 0_{N \times (N_f - N)} \\ 0_{(N_f - N) \times N} & M_0 \end{pmatrix}, \quad \frac{q^\dagger}{\mu^*} = \frac{\tilde{q}}{\mu} = \begin{pmatrix} \mathbf{1}_N \\ 0_{(N_f - N) \times N} \end{pmatrix}, \quad (2.7)$$

where M_0 is an arbitrary $N_f - N$ by $N_f - N$ matrix. Except for the first condition in equation (2.6), all the other conditions are satisfied. So the vacuum energy is given by

$$V = |h\mu^2|^2 (N_f - N) > 0, \quad (2.8)$$

¹When the gauge group is $U(N)$ instead of the $SU(N)$ (i.e. when we gauge the $U(1)_B$ with its coupling put equal to that of the $SU(N)$), the second term in V_D is not necessary.

and thus the vacuum spontaneously breaks the $\mathcal{N} = 1$ supersymmetry. We will see this supersymmetry breaking from the view point of the D-brane configuration in section 3.1.

Notice that this vacuum shows color-flavor locking which is invariant under the $SU(N)_{c+f}$ global transformation,

$$M \rightarrow U M U^\dagger = M, \quad q \rightarrow g q U^\dagger = q, \quad \tilde{q} \rightarrow U \tilde{q} g^\dagger = \tilde{q}. \quad (2.9)$$

Here M, q, \tilde{q} are given in (2.7), $g \in SU(N)$ is a global rotation of the gauge group $SU(N)$, and U is an element of $SU(N_f)$ defined by

$$U = \begin{pmatrix} g & \\ & \mathbf{1}_{N_f-N} \end{pmatrix} \in SU(N_f). \quad (2.10)$$

The supersymmetry-breaking vacua given in (2.7) have several flat directions. A part of them are truly massless Nambu-Goldstone modes associated with the spontaneously broken global symmetries, while the others are classical pseudo-moduli fields which acquires positive masses by one-loop contributions to the effective potential [2]. The vacuum of the potential is then $M_0 = 0$ and the Nambu-Goldstone modes are generated by

$$q = (\mu \mathbf{1}_N, 0) \rightarrow \hat{g} (\mu \mathbf{1}_N, 0) \hat{U}^\dagger, \quad (2.11)$$

where $\hat{g} \in SU(N)$ and $\hat{U} \in U(N_f) \sim SU(N_f) \times U(1)_B$. We will study in detail the vacuum manifold of this magnetic theory in the following subsections.

In the SQCD above, the masses of the quarks are chosen to be the same, which are proportional to μ^2 . We can introduce different mass for each quark, by replacing the superpotential (2.2) by

$$W = h \text{Tr}_c [q M \tilde{q}] - h \text{Tr}_f [m M], \quad (2.12)$$

where $m = \text{diag}(m_1, m_2, \dots, m_{N_f})$, and m_i is the quark mass of the SQCD times a dynamical scale $(-\hat{\Lambda})$, see [2]. We can bring m_i real and positive classically, and choose $m_1 \geq m_2 \geq \dots \geq m_{N_f}$. When all the masses are non-vanishing, the supersymmetry is broken at the vacuum. However, when $m_{N+1} = \dots = m_{N_f} = 0$, the rank condition is satisfied, and the supersymmetry is unbroken at the vacuum although the vacuum expectation value is the same as that given in equation (2.7). Notice that, by the mass arrangement $m = \text{diag}(\underbrace{\mu^2, \dots, \mu^2}_N, \underbrace{0, \dots, 0}_{N_f-N})$, the flavor symmetry $SU(N_f)$ is explicitly broken down to $SU(N)$.

2.2 Topological solitons

Let us study the possible existence of the topological solitons in the meta-stable vacua in the magnetic theory. If there are topological solitons, the corresponding solitons in the meta-stable vacua in the SQCD have lifetime of the order of the lifetime of the meta-stable vacua.

We also study the moduli space \mathcal{M}_{vac} of the meta-stable vacua and the topological solitons in the nonlinear sigma model whose target spaces is the moduli space. These

nonlinear sigma models are obtained as the effective action for the energy lower than the mass scale of the pseudo Nambu-Goldstone modes. It is important to note that there is no obstruction to continuously deform the configuration corresponding to the nonlinear sigma model soliton to the meta-stable vacua, in the magnetic theory. Moreover, this deformation is not related to the true supersymmetric vacua and is closed in the region near the meta-stable vacua. Thus the nonlinear sigma model solitons have a relatively short lifetime of the order of the scale given by the mass of the pseudo Nambu-Goldstone modes (if they do not correspond to any topological soliton of the magnetic theory of the SQCD).

In advance, let us list our results here. In the magnetic theory of the SQCD with the gauge group $SU(N_c)$, the gauge group $SU(N)$ is completely broken at the meta-stable vacuum. Since $\pi_k(SU(N)) = 0$ for $0 \leq k \leq 2$, we expect there is no topological soliton except (constrained) instantons in its Euclideanized theory. In fact, we find the homotopy groups of the moduli space of vacua, as

$$\pi_0(\mathcal{M}_{\text{vac}}) = \pi_1(\mathcal{M}_{\text{vac}}) = \pi_2(\mathcal{M}_{\text{vac}}) = 0. \tag{2.13}$$

Thus we expect no topological solitons (global monopole / vortex string / domain wall) in the meta-stable vacua, and there is no string or domain wall even in the sigma model which is the low energy effective theory.²

In order to have non-trivial solitons, we need to consider $U(N_c)$ gauge group instead of the $SU(N_c)$, in other words, gauging the $U(1)_B$ symmetry. This is equivalent to considering $SU(N_c) \times U(1)_B / \mathbf{Z}_N$ gauge group in the magnetic theory.³ In this case, the gauge group $SU(N) \times U(1)_B$ is completely broken at the meta-stable vacuum and then we expect to have (semi-)local vortex strings from the broken $U(1)_B$. This is the meta-stable string in the SQCD with the gauged $U(1)_B$. The moduli space of vacua $\widetilde{\mathcal{M}}_{\text{vac}}$ has the following homotopy groups:

$$\pi_0(\widetilde{\mathcal{M}}_{\text{vac}}) = \pi_1(\widetilde{\mathcal{M}}_{\text{vac}}) = 0, \quad \pi_2(\widetilde{\mathcal{M}}_{\text{vac}}) = \mathbf{Z}. \tag{2.14}$$

Therefore the gauged $U(1)$ allows us to have global monopoles and sigma model strings (lumps). The global monopoles are usually forbidden since they have infinite mass. The sigma model strings correspond to the (semi-)local vortex strings in the magnetic theory.

When the gauge group is $SO(N_c)$, the condition to have the ultra-violet SQCD is $N_f < (3/2)(N_c - 2)$. The gauge group of the dual magnetic theory is $SO(N)$ with $N =$

²In this paper we do not consider the particle like solitons in the sigma model although $\pi_3(\mathcal{M}_{\text{vac}})$ is not always trivial. The reasons are as follows. The Derrick's theorem states that there are no stable sigma model solitons whose co-dimension is more than 1. The theorem is valid with no higher derivative terms, so a smooth sigma model soliton might exist if we include higher derivative corrections, like skyrmions. However, this corresponds to going to higher energy regime beyond the non-linear sigma model limit. We do not have any natural reason why one can abandon pseudo moduli and higher excited modes while keeping the particular higher derivative terms on the moduli fields. Even when one assumes such truncation, the lifetime of the sigma model soliton is quite short compared to the lifetime of the meta-stable vacua. We would like to thank K. Intriligator on this point.

³Since the gauge couplings of the $SU(N_c)$ and the $U(1)_B$ are generically different, the $SU(N_c) \times U(1)_B / \mathbf{Z}_N$ is not equivalent to $U(N)$ although they are topologically equivalent. Note that $\mathbf{Z}_N \in SU(N)$ and $\mathbf{Z}_N \in U(1)_B$ act in the same way on the matter fields in the magnetic theory.

$N_f - N_c + 4 \geq 1$. For the breaking of the gauge group, we have $\pi_1(\text{SO}(N)) = \mathbf{Z}_2$ except for the case of $\text{SO}(2) \sim \text{U}(1)$ where $\pi_1(\text{SO}(2)) = \mathbf{Z}$. For the moduli space, we find

$$\begin{aligned} \pi_0(\mathcal{M}_{\text{SO}}) &= 0, \\ \pi_1(\mathcal{M}_{\text{SO}}) &= \begin{cases} \mathbf{Z} & (\text{for } (N_f, N_c) = (2, 5)) \\ 0 & (\text{for the other cases}) \end{cases} \\ \pi_2(\mathcal{M}_{\text{SO}}) &= \begin{cases} \mathbf{Z} & (\text{for } N_c = N_f + 2, \text{ except } N_f = 4) \\ \mathbf{Z}_2 & (\text{for the other cases}) \end{cases} \end{aligned} \quad (2.15)$$

Thus, when $N = 2$ (or equivalently $N_c = N_f + 2$), the gauge group is $\text{SO}(2) \sim \text{U}(1)$ which is broken, thus there are semilocal strings.⁴ For generic N we find \mathbf{Z}_2 solitonic strings.⁵ These strings are topological solitons, hence are meta-stable in the original SQCD.

In the rest of this section, first we shall obtain the moduli space of the meta-stable vacua, with the un-gauged/gauged $\text{U}(1)_B$ symmetry for the $\text{SU}(N_c)$ gauge groups. Then we will derive the homotopy groups (2.13), (2.14) and (2.15). The case with the $\text{SO}(N_c)$ will be described briefly. In the next section, we study the brane realization of the (semi-)local vortex strings for the case of the gauged $\text{U}(1)_B$, and in section 4 we explicitly construct classical solutions of the vortex strings.

2.3 Moduli space of vacua

The global symmetries of the magnetic theory of the SQCD are $\text{SU}(N_f) \times \text{U}(1)_B$, where the $\text{U}(1)_B$ symmetry acts on the overall phase of the field q . The local symmetry is $\text{SU}(N)$. We will deal with the case of the gauged $\text{U}(1)_B$ and $\text{SO}(N)$ gauge group later.

The moduli space of the vacua is defined as a quotient space G/H , where G is the global symmetry of the theory, and H is global symmetries which leave the vacuum invariant. Obviously in our case

$$G = \text{U}(N_f). \quad (2.16)$$

Note that when we wrote the global symmetries as $\text{SU}(N_f) \times \text{U}(1)_B$, we were not precise concerning the discrete subgroup, and in fact $\text{U}(N_f) = (\text{SU}(N_f) \times \text{U}(1)_B)/\mathbf{Z}_{N_f}$ is the correct global symmetry of the theory.⁶ Let us look for the group H . Consider the following elements of a subgroup $S(\text{U}(N) \times \text{U}(N_f - N))$ of the $\text{SU}(N_f)$:

$$U = \begin{pmatrix} e^{i\frac{\theta}{N}} g_N & \\ & e^{-i\frac{\theta}{N_f - N}} g_{N_f - N} \end{pmatrix} \quad (2.17)$$

where we have defined $g_M \in \text{SU}(M)$. Consider the $S[\text{U}(1) \times \text{U}(1)]$ factor by choosing $g_N = \mathbf{1}_N$ and $g_{N_f - N} = \mathbf{1}_{N_f - N}$,

$$\tilde{U} = \begin{pmatrix} e^{i\frac{\theta}{N}} \mathbf{1}_N & \\ & e^{-i\frac{\theta}{N_f - N}} \mathbf{1}_{N_f - N} \end{pmatrix}. \quad (2.18)$$

⁴For the special case $N_f = 4$, see the next subsection.

⁵For the special case of $(N_f, N_c) = (2, 5)$, sigma-model domain walls / global strings are possible.

⁶In the $\text{U}(N_f)$, elements $\exp[2\pi i n/N_f] \mathbf{1}_{N_f}$ ($n = 0, 1, 2, \dots, N_f - 1$) belong to both the $\text{SU}(N_f)$ and the $\text{U}(1)_B$.

We call this $\tilde{U}(1)$ symmetry. The other $U(1)_B$ symmetry together with this $\tilde{U}(1)$ act on the vacuum as

$$\mu(\mathbf{1}_N, 0) \rightarrow \mu e^{i\theta_B}(\mathbf{1}_N, 0) \tilde{U}^\dagger = \mu \left(e^{i(\theta_B - \frac{\theta}{N})} \mathbf{1}_N, 0 \right). \quad (2.19)$$

Therefore, when we have a relation

$$N\theta_B = \theta \quad (2.20)$$

the vacuum is invariant. The combined symmetry is written as an element of $U(N_f)$,

$$U' = \begin{pmatrix} \mathbf{1}_N & \\ & e^{-i\frac{\theta}{N_f-N}} \mathbf{1}_{N_f-N} \end{pmatrix}. \quad (2.21)$$

We call this $U'(1)$ symmetry.

The element g_N in the upper-left block in (2.17), when it acts on the vacuum, can be absorbed by the $SU(N)$ gauge symmetry which acts on q from the left hand side. Thus, $SU(N)$ is still a symmetry of the vacuum: this is the color-flavor locking. Furthermore, the element g_{N_f-N} in the bottom-right block in (2.17) is just an isotropy for the vacuum. Noting that together with the above $U'(1)$, this $SU(N_f - N)$ is upgraded to form a $U(N_f - N)$, after considering the discrete subgroup Z_{N_f-N} properly. Therefore, in total, the remaining global symmetry of the vacuum is

$$H = SU(N) \times U(N_f - N). \quad (2.22)$$

The moduli space of the vacua is

$$\mathcal{M}_{\text{vac}} = \frac{U(N_f)}{SU(N) \times U(N_f - N)}. \quad (2.23)$$

Next, let us gauge the $U(1)_B$ symmetry. If we gauge $U(1)_B$, the gauge group of the magnetic theory becomes $U(N)$.⁷ It is easy to see that the meta-stable vacua (2.7) are still meta-stable vacua for the gauged $U(1)_B$ case. The total global symmetry G is in this case $G = SU(N_f)$. The remaining global symmetry H is the same as before, but we can write it as $H = S[U(N_f - N) \times U(N)] = SU(N_f - N) \times SU(N) \times \tilde{U}(1)$, since $U(1)_B$ is gauged. Consequently, the vacuum manifold is given by

$$\begin{aligned} \tilde{\mathcal{M}}_{\text{vac}} &= \frac{SU(N_f)}{S[U(N) \times U(N_f - N)]} = \frac{SU(N_f)}{SU(N) \times SU(N_f - N) \times \tilde{U}(1)} \\ &= Gr_{N_f, N} \end{aligned} \quad (2.24)$$

which is a Grassmanian manifold.

We can find the moduli space \mathcal{M}_{vac} easier by considering gauge invariant operators. Here the gauge invariant operators are the meson M_i^j and baryons $b_{i_1 i_2 \dots i_N}, \tilde{b}_{i_1 i_2 \dots i_N}$ where

⁷Of course, this makes the original SQCD asymptotically non-free. However, as a cut-off theory it could be useful for applications of the meta-stable vacua to cosmologies and phenomenological model constructions.

i_k runs from 1 to N_f . Actually, the global symmetry $U(N_f) \sim SU(N_f) \times U(1)_B$ was broken by the gauge invariant operators $b_{12\dots N} \sim \mu \epsilon_{12\dots N} \sim \tilde{b}_{12\dots N}$, the other components of the b and \tilde{b} vanish, and $M_i^j = 0$. Therefore, we find again that the unbroken global symmetry is $SU(N) \times U(N_f - N)$ and the moduli space \mathcal{M}_{vac} is indeed (2.23), because the moduli space is spanned by massless scalars while we know that the massless scalars in the meta-stable vacua are only the Nambu-Goldstone modes associated with the global symmetry breaking [2]. For the gauged $U(1)_B$ case, the global symmetry becomes $SU(N_f)$ and the baryon b is not a gauge invariant operator, but $b\tilde{b}$ is. Thus, the unbroken global symmetry group in this case is $S(U(N) \times U(N_f - N))$ and the moduli space $\tilde{\mathcal{M}}_{\text{vac}}$ is indeed given by (2.24). In the limit of the vanishing $U(1)_B$ gauge coupling $e \rightarrow 0$, the scalar fields which are massive due to the super Higgs mechanism become massless, which change $\tilde{\mathcal{M}}_{\text{vac}}$ to \mathcal{M}_{vac} . However, this is globally nontrivial because $\mathcal{M}_{\text{vac}} \neq \tilde{\mathcal{M}}_{\text{vac}} \times (S^1 \text{ of } U(1)_B)$.

Finally, we briefly mention the moduli space of the meta-stable vacua in the massive SQCD with the $SO(N_c)$ gauge group, under a condition $0 < N_c - 4 < N_f < \frac{3}{2}(N_c - 2)$. The gauge group of the magnetic theory is $SO(N)$ ($N = N_f - N_c + 4$), and the moduli space is

$$\mathcal{M}_{\text{SO}} = \frac{SO(N_f)}{SO(N) \times SO(N_f - N)}. \tag{2.25}$$

This is obtained in quite a similar way, since the structure of the superpotential is the same. The $SO(N)$ appearing in the denominator is from the gauge symmetry locked with a part of the global symmetry, thus the vacua are in the color-flavor locking phase.

2.4 Homotopy groups

Homotopy groups directly indicate the existence of global (or sigma model) solitons, and we here evaluate the homotopy groups of the vacuum manifold (2.23), (2.24) and (2.25).

First, we consider the case of the un-gauged $U(1)_B$ symmetry, (2.23). The homotopy exact sequence [14] concerning the vacuum manifold $\mathcal{M}_{\text{vac}} = G/H$ is

$$0 = \pi_2(G) \xrightarrow{f_1} \pi_2(G/H) \xrightarrow{f_2} \pi_1(H) \xrightarrow{f_3} \pi_1(G) \xrightarrow{f_4} \pi_1(G/H) \xrightarrow{f_5} \pi_0(H) = 0. \tag{2.26}$$

Here we have used the fact that $\pi_2(G) = \pi_2(U(N_f)) = 0$ and $\pi_0(H) = \pi_0(SU(N) \times U(N_f - N)) = 0$. We know that $\pi_1(U(M)) = \mathbf{Z}$ for any $M \geq 1$, so the exact sequence is written as

$$0 \xrightarrow{f_1} \pi_2(G/H) \xrightarrow{f_2} \mathbf{Z} \xrightarrow{f_3} \mathbf{Z} \xrightarrow{f_4} \pi_1(G/H) \xrightarrow{f_5} 0. \tag{2.27}$$

To obtain the homotopy groups $\pi_i(G/H)$ for seeing the solitons, it is necessary to know how the map $\mathbf{Z} \rightarrow \mathbf{Z}$ is organized. First, let us see how the $\pi_1(G) = \mathbf{Z}$ is generated. In the $G = U(N_f)$, consider the following loop $u(t)$ where t is parameterizing the loop as $0 \leq t \leq 1$ and $u(0) = u(1)$:

$$u(t) = e^{4\pi it/N_f} \mathbf{1}_{N_f} \in U(1)_B \quad (0 \leq t \leq 1/2) \tag{2.28}$$

$$u(t) \in SU(N_f) \quad (1/2 \leq t \leq 1) \tag{2.29}$$

This is a non-trivial loop going from the origin, through $e^{\pi i/N_f} \mathbf{1}_{N_f}$, back to the origin. In the same manner, $\pi_1(H) = \mathbf{Z}$ is generated by a loop $u'(t)$ in the $U(N_f - N)$, whose definition is completely analogous to $u(t)$. The map $\mathbf{Z} \rightarrow \mathbf{Z}$ is determined by the relation between this $u'(t)$ and $u(t)$. Note that $u'(t)$ ($0 \leq t \leq 1/2$) is decomposed into a product of $U(1)_B$ and $SU(N_f)$ as

$$u'(t) = \begin{pmatrix} \mathbf{1}_N & \\ & e^{4\pi i t/(N_f - N)} \mathbf{1}_{N_f - N} \end{pmatrix} = e^{4\pi t/N_f} \begin{pmatrix} e^{-4\pi i t/N} \mathbf{1}_N & \\ & e^{4\pi i t/(N_f - N)} \mathbf{1}_{N_f - N} \end{pmatrix}.$$

Looking at the $U(1)_B$ factor in the last expression, we find that the loop $u'(t)$ reaches the point $e^{2\pi t/N_f}$ at $t = 1/2$. This is the same point as $u(1/2)$. Therefore, the loop $u(t)$ rounds once when the loop $u'(t)$ rounds once: the map $\mathbf{Z} \rightarrow \mathbf{Z}$ is one-to-one and on-to.

Using this fact, we can obtain the homotopy groups of the quotient space, using the exactness of the sequence. First, using $\text{Ker}(f_3) = 0$, we have $\text{Im}(f_2) = 0$, which implies $\text{Ker}(f_2) = \pi_2(G/H)$. Again the exactness means $\text{Ker}(f_2) = \text{Im}(f_1) = 0$, thus we obtain $\pi_2(G/H) = 0$. On the other hand, since $\text{Im}(f_3) = \mathbf{Z}$, we have $\text{Ker}(f_4) = \mathbf{Z}$. Hence $\text{Im}(f_4) = 0$, which leads to $\text{Ker}(f_5) = 0$. However, f_5 is the last element in the exact sequence, so $\pi_1(G/H) = \text{Ker}(f_5)$. Thus we obtain $\pi_1(G/H) = 0$. This is the proof of the result (2.13).

The moduli space of the theory with the gauged $U(1)_B$ symmetry has a non-trivial homotopy,

$$\pi_2 \left(\frac{SU(N_f)}{SU(N_f - N) \times SU(N) \times \tilde{U}(1)} \right) = \pi_1 (SU(N_f - N) \times SU(N) \times \tilde{U}(1)) = \mathbf{Z}. \quad (2.30)$$

We have used a well-known homotopy formula which is accessible in this case.

Finally, for the gauge group $SO(N_c)$, we write the moduli space (2.25) as G/H where $G = SO(N_f)/SO(N_f - N)$ is a Stiefel manifold and $H = SO(N)$. To obtain the homotopy groups of G/H , we use the homotopy groups of G : $\pi_{i < N_f - N}(G) = 0$, $\pi_{N_f - N}(G) = \mathbf{Z}$ (for even $N_f - N$ or $N = 1$), and $\pi_{N_f - N}(G) = \mathbf{Z}_2$ (for odd $N_f - N$ and $N > 1$). Then, the exact sequence of the following generic form

$$0 = \pi_i(G) \rightarrow \pi_i(G/H) \rightarrow \pi_{i-1}(H) \rightarrow \pi_{i-1}(G) = 0 \quad (2.31)$$

leads to a generic formula for homotopy groups of G/H (a similar derivation for $O(N_f)$ group can be found in [14]). But when i is large or N_f is small, the end points of the above exact sequence do not vanish, and special treatment is required. The results are already listed at the end of section 2.2. In addition, for example, one can find $\pi_3(G/H) = 0$ for $N > 3$ or $N_f - N > 3$.

3. Brane realization of vortex strings in meta-stable vacua

In this section, we show that classically there exist solitonic strings in the magnetic side of the SQCD, by studying corresponding brane configurations.

In section 3.1, we first review the brane realization of the meta-stable vacua [3–5]. Then, in section 3.2, we review the brane realizations of the $\mathcal{N} = 2$ supersymmetric gauge

NS	1	2	3	-	-	-	-	8	9
NS'	1	2	3	4	5	-	-	-	-
D6	1	2	3	-	-	-	7	8	9
D4	1	2	3	-	-	6	-	-	-

Table 1: Hanany-Witten setup for the $\mathcal{N} = 1$ supersymmetric gauge theory.

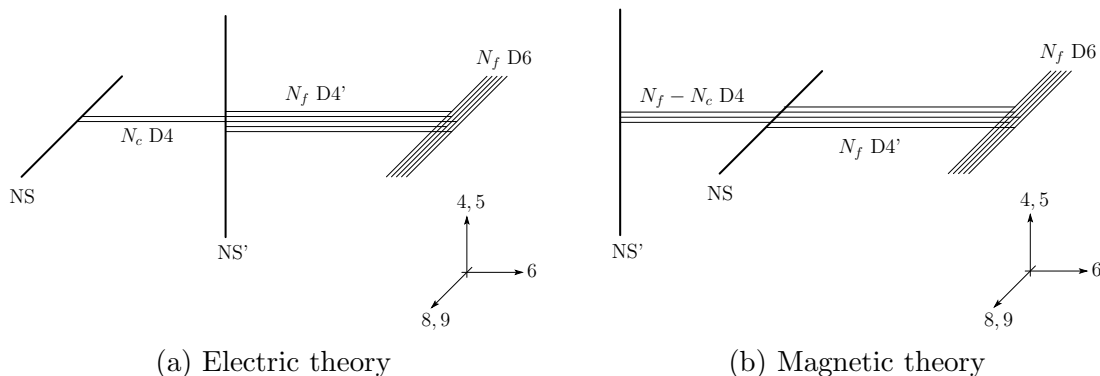


Figure 1: Brane configuration for massless SQCD.

theories and their solitonic strings. In fact, in section 3.3, we will find a useful analogy between the solitonic strings in the supersymmetry-breaking meta-stable vacua and the well-known BPS solitonic strings in supersymmetric vacua of the $\mathcal{N} = 2$ supersymmetric gauge theory. There we construct brane configurations corresponding to the solitonic strings in the magnetic dual of the massive SQCD. Various properties of the solitonic strings are predicted from string theory.

3.1 Review: brane realization of meta-stable vacua in SQCD

The brane configurations in the type IIA string theory can capture well the properties of both the electric and the magnetic theories of the SQCD.⁸ The Hanany-Witten setup [9] for the $\mathcal{N} = 1$ SQCD [8] (see [10] for an extensive review) consists of two NS5-branes, N_f D6-branes and N_c D4-branes (N_f D4'-branes) whose world-volume orientations are summarized in table 1. The SQCD is realized on the D4-brane worldvolume at low energy. The brane configuration for the massless SQCD is depicted in figure 1. The electric theory is realized in figure 1(a). The $U(N_c)$ vector multiplet corresponds to the spectrum of a fundamental string ending on the N_c D4-branes which are suspended between the NS5-brane and the NS'5-brane, while the chiral multiplets Q and \tilde{Q} come from a fundamental string stretched between the N_c D4-branes and the N_f D4'-branes. The dual (magnetic) theory can be obtained by exchanging the positions of NS5-brane and NS'5-brane, for example, on the x^6 axis, see figure 1(b). Hence, the $U(N)$ vector multiplet appears from a fundamental

⁸The brane configurations are valid for analyzing the meta-stable vacua only in the limit $g_s \rightarrow 0$ [15]. In this paper, however, we will concentrate on classical solitons in the meta-stable vacua, thus brane configurations are helpful.

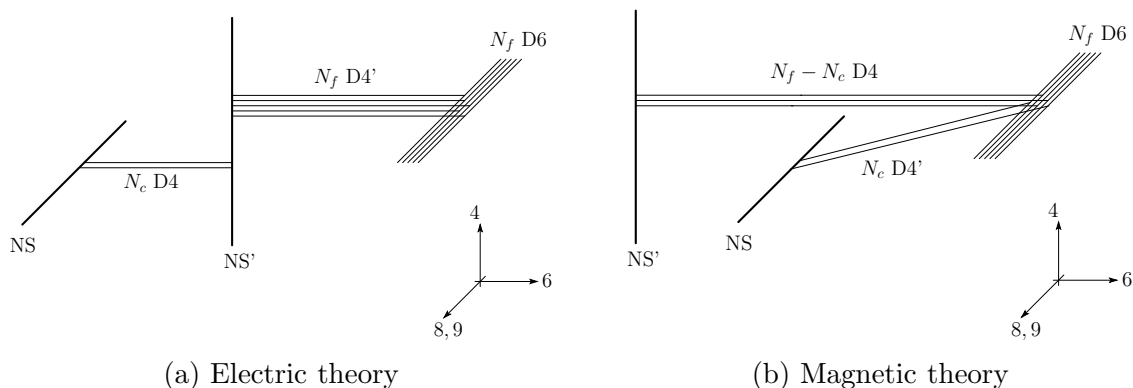


Figure 2: Brane configuration for massive SQCD.

string between the $N = N_f - N_c$ D4-branes, and the dual quarks q and \tilde{q} come from fundamental strings between the D4-branes and the D4'-branes. Furthermore, the meson field M appears from fundamental strings on the N_f D4'-branes, which corresponds to a massless degree of freedom for the transverse motion of the D4'-branes for the x^8 and the x^9 directions.

The brane configuration for the electric theory of the massive SQCD (the masses are real) is obtained just by parallelly shifting the D4'-branes along the x^4 axis away from the D4-branes, as shown in figure 2(a). Minimum length of a fundamental string stretched between the D4-branes and the D4'-branes is the distance between them. This non-zero distance leads to nonzero masses for the quark fields Q and \tilde{Q} . Let us next turn to the dual theory of the massive SQCD. The brane configuration of it can provide an intuitive and good understanding of the dual theory [3–5]. We start with figure 1(b) and we lift the N_f D6-branes parallelly upward along the x^4 axis. Then the $N (= N_f - N_c)$ D4-branes and the same number of D4'-branes in figure 1(b) are joined together and pulled by the D6-branes, then lifted upward. On the other hand, remaining $N_c (= N_f - N)$ D4'-branes cannot make a pair with any of the D4-branes, so they are still stretched between the NS5-brane and the D6-branes, see figure 2(b) [3–5]. Obviously, the brane configuration in figure 2(b) breaks the bulk supersymmetries completely, and it is nothing but the supersymmetry-breaking meta-stable vacua. Comparing figure 1(b) and figure 2(b), the length of the $N_f - N (= N_c)$ D4-branes in the former longer than that in the latter. This difference is regarded as the potential energy of the supersymmetry-breaking meta-stable vacua, and it agrees with (2.8).

As described in the last of section 2.1, we may introduce various quark masses. In this case, the position of the D6-branes in the x^4 - x^5 plane is specified by m_i . The situation of the supersymmetry restoration at the vacuum, described in section 2.1, can be easily understood in the brane configuration. When $m_{N+1} = \dots = m_{N_f} = 0$, both the NS5-brane and the N_c D6-branes sit at the origin of the x^4 - x^5 plane, thus the N_c D4'-branes connecting them are aligned parallel to the remaining N D4-branes. Thus the bulk supersymmetries are not completely broken.

NS	1	2	3	4	5	-	-	-	-
NS'	1	2	3	4	5	-	-	-	-
D6	1	2	3	-	-	-	7	8	9
D4	1	2	3	-	-	6	-	-	-
D2	-	-	3	-	-	-	-	-	9

Table 2: Hanany-Witten setup for the $\mathcal{N} = 2$ $U(N)$ gauge theory. The D2-brane is added to describe the solitonic strings in the theory.

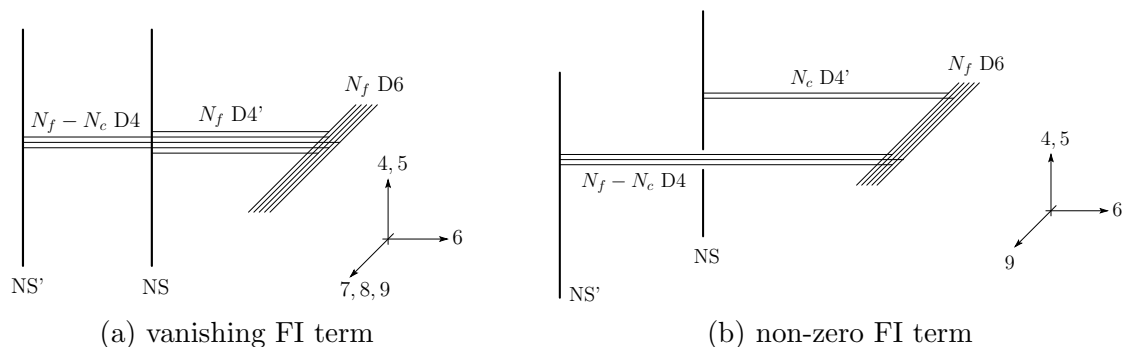


Figure 3: Brane realization of the 4d $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theories.

3.2 Review: $\mathcal{N} = 2$ supersymmetric gauge theory and vortex strings

In this subsection we review the brane configuration of BPS vortex strings in $\mathcal{N} = 2$ non-Abelian gauge theory [11] (see also [12] for brane configurations of vortices in Abelian gauge theory). This in fact turns out to be quite helpful for constructing a brane configuration of vortex strings in the supersymmetry-breaking meta-stable vacua in the next subsection.

Brane realization and field theory vacua. We start with the $\mathcal{N} = 2$ Hanany-Witten set up, whose brane orientations are summarized in table 2. We consider the $\mathcal{N} = 2$ 4d supersymmetric $U(N)$ gauge theory on N D4-branes suspended between a NS5-brane and a NS'5-brane, see figure 3(a). The fields which appear in the gauge theory are hypermultiplets including scalar components of dual quarks q and \tilde{q} , and a vector multiplet including a complex scalar field Σ in the adjoint representation of the gauge group $U(N)$. Here q and \tilde{q} appear as in the same way as the magnetic theory of the SQCD. The new field Σ comes from fundamental-strings whose end points are attached on the N D4-branes and it corresponds to transverse motion of the D4-branes along the x^4 and x^5 direction. On the other hand, the meson field M in the magnetic theory of the SQCD does not appear here, because the N_f D4'-branes cannot move freely in this situation.

We can turn on a Fayet-Iliopoulos (FI) parameter v^2 while keeping the $\mathcal{N} = 2$ supersymmetries. This is necessary to have BPS solitonic strings. In the brane configuration, the FI term is realized as a parallel transport of the NS5-brane in the space of x^7, x^8, x^9 . As an example, we parallel-transport the NS5-brane along the x^9 axis,⁹ see figure 3(b).

⁹The rotation in the x^7, x^8, x^9 space corresponds in the field theory to the $SU(2)_R$ symmetry. Namely,

The superpotential of this field theory is quite similar to that given in equation (2.2) of the magnetic theory of the SQCD:

$$W = g\mathrm{Tr}_c [q\Sigma\tilde{q}] - v^2\mathrm{Tr}_c\Sigma. \quad (3.1)$$

With this FI parameter v^2 , as is obvious in figure 3(b), the D4-branes cannot move freely. This fact can be seen consistently in the field theory, by looking at the scalar potentials

$$V_F = \mathrm{Tr}_c \left\{ g^2 |q\tilde{q} - v^2\mathbf{1}_N|^2 + |q\Sigma|^2 + |\tilde{q}^\dagger\Sigma^\dagger|^2 \right\}, \quad (3.2)$$

$$V_D = \mathrm{Tr}_c \left\{ \frac{g^2}{4} (qq^\dagger - \tilde{q}^\dagger\tilde{q})^2 - \frac{1}{g^2} [\Sigma, \Sigma^\dagger]^2 \right\}. \quad (3.3)$$

These scalar potentials are almost the same as those in the SQCD given in equations (2.4) and (2.5). The only difference is the size of the field Σ ($N \times N$) and the field M ($N_f \times N_f$), and consequently the rank condition. The classical vacuum of this $\mathcal{N} = 2$ model is in the Higgs phase:

$$\Sigma = 0, \quad qq^\dagger - \tilde{q}^\dagger\tilde{q} = 0, \quad q\tilde{q} = v^2\mathbf{1}_{N_c}. \quad (3.4)$$

There is no flat direction in Σ at the classical level on the contrary to the case of the $\mathcal{N} = 1$ SQCD. Furthermore, the Higgs branch is well-known to be a cotangent bundle over a complex Grassmanian manifold

$$T^*Gr_{N_f, N} = T^* \left[\frac{\mathrm{SU}(N_f)}{\mathrm{SU}(N) \times \mathrm{SU}(N_c) \times \tilde{U}(1)} \right]. \quad (3.5)$$

The base space Grassmanian is parameterized by $q = \tilde{q}^\dagger$ similarly to the case of the $\mathcal{N} = 1$ SQCD. In this case the rank condition is satisfied because the rank of $q\tilde{q}$ is the same as $v^2\mathbf{1}_N$, so that the vacuum energy vanishes. Thus the vacua maintain the full supersymmetries.

1/2 BPS solitonic strings. There are 1/2 BPS solitonic strings (vortex strings) in this $\mathcal{N} = 2$ theory. They are called semilocal non-Abelian vortex strings. The Abelian version has been known for decades [13], while its non-Abelian extension of our concern has been considered in [11] (see also [16–18]). The non-Abelian semilocal string is a natural extension of the well-known Abrikosov-Nielsen-Olesen (ANO) vortex in the Abelian-Higgs model. The 1/2 BPS equation for the vortex is

$$(D_1 \pm iD_2)\phi = 0, \quad F_{12} \pm \frac{g^2}{2} (2v^2 - \phi\phi^\dagger) = 0, \quad (3.6)$$

where we have assumed that all the fields depend only on x^1 and x^2 , and

$$q = \tilde{q}^\dagger \equiv \frac{\phi}{\sqrt{2}}, \quad \Sigma = A_0 = A_3 = 0, \quad (3.7)$$

the FI parameters are transformed as a triplet under the $\mathrm{SU}(2)_R$. In this paper, we assume that the parallel-transport of the NS5-brane along the x^7 axis corresponds to the FI D -term while that along x^8, x^9 axis to for the FI F -term.

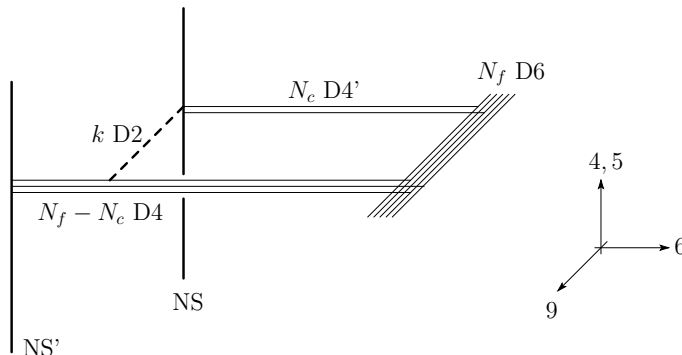


Figure 4: 1/2 BPS semilocal strings in $\mathcal{N} = 2$ $4d$ super Yang-Mills-Higgs system.

which are consistent with all the equations of motion. The tension (the energy per unit length along x^3) of the system is bounded from below by the topological charge $k \in \pi_1(\text{U}(N)) = \mathbf{Z}$, and the bound is saturated by any solution of the 1/2 BPS equations (3.6),

$$\mathcal{E} = \mp 2v^2 \int d^2x F_{12} = 4v^2 \pi |k|. \tag{3.8}$$

As mentioned before, the vacuum shows the color-flavor locking, namely the symmetry of the vacuum is $\text{SU}(N)_{c+f}$. When a minimal vortex sits in the vacuum, the symmetry is spontaneously broken to $S[\text{U}(N-1) \times \text{U}(1)]$, so the moduli space of the vortex includes in particular an “orientational” moduli $\mathbf{C}P^{N-1}$, in addition to the position moduli \mathbf{C} . This $\mathbf{C} \times \mathbf{C}P^{N-1}$ is the total moduli space for the BPS non-Abelian strings with $N = N_f$, but the 1/2 BPS semilocal vortex strings ($N_f > N$) have additional moduli parameters concerning the size of the vortices.

This non-Abelian semilocal string can be realized in the brane configuration, which was found by Hanany and Tong [11]. The soliton with the co-dimension 2 (compared to the D4-branes) is k D2-branes which are suspended between the NS’5-brane and the $(N_f - N_c)$ D4-branes, as shown in figure 4. The worldvolume of the D2-branes is along x^0, x^1 and x^2 , as in the table 2. The tension of the D2-branes is proportional to the distance between the NS5-brane and the NS’5-brane (along the x^9 axis), namely the amount of the FI term $2v^2$. This is consistent with the field theory result in equation (3.8).

A topological property of the moduli space for these 1/2 BPS non-Abelian semilocal strings is captured from massless excitations of fundamental strings on k D2-branes, N D4-branes and N_c D4’-branes [11]. Let us denote k by k matrix Z for expected zero modes between D2-D2, k by N matrix ψ for those between D2-D4 and N_c by k matrix $\tilde{\psi}$ for those between D2-D4’. Then the moduli space of the solitonic strings is given by the following Kähler quotient:

$$\left\{ \left[Z, Z^\dagger \right] + \psi \psi^\dagger - \tilde{\psi}^\dagger \tilde{\psi} \propto \mathbf{1}_N \right\} / \text{U}(k). \tag{3.9}$$

For a single vortex, Z is just a complex constant. Then, if we look at $\tilde{\psi} = 0$ sector, this quotient gives us the $\mathbf{C} \times \mathbf{C}P^{N-1}$ which is consistent with the above field theory result.

NS	1	2	3	-	-	-	-	8	9
NS'	1	2	3	4	5	-	-	-	-
D6	1	2	3	-	-	-	7	8	9
D4	1	2	3	-	-	6	-	-	-
D2	-	-	3	-	-	-	7	-	-

Table 3: The Worldvolumes of the branes for the massless $\mathcal{N} = 1$ SQCD and its dual. We added the D2-brane which represents a vortex string.

The 1/2 BPS non-Abelian semilocal strings have three kinds of moduli parameters: (i) positions (ii) orientations (iii) sizes. Roughly speaking, Z corresponds to the positions, ψ to the orientations and $\tilde{\psi}$ to the sizes.

3.3 Brane realization of vortex strings in meta-stable vacua

We have seen that 1/2 BPS semilocal strings naturally appear in the $\mathcal{N} = 2$ supersymmetric gauge theory with the FI parameter, and their corresponding D-brane picture is well understood. In this subsection, we apply the idea to the magnetic theory of the SQCD. We deal with two cases: magnetic theory of (i) massless $\mathcal{N} = 1$ SQCD with an analogous FI term, and of (ii) massive $\mathcal{N} = 1$ SQCD. We find D-branes corresponding to the semilocal strings, which shows the existence of the solitonic strings in the theories.

1/2 BPS solitonic strings in magnetic theory of massless SQCD. The brane configurations are quite useful to find out possible solitonic defects, as reviewed in the previous subsection. Let us study what is a possible introduction of the D2-brane in the brane configuration of the magnetic theory of the massless $\mathcal{N} = 1$ SQCD, figure 1. It is clear that we cannot attach a D2-brane in this brane configuration. But as is suggested from the $\mathcal{N} = 2$ example in the previous subsection, if we introduce a FI term, we obtain a stable vortex string, as we shall see below.

In the previous $\mathcal{N} = 2$ case, we have three candidates, x^7, x^8 and x^9 , as a possible direction along which we can parallel-transport the NS'5-brane. In the present case of the magnetic theory of the massless $\mathcal{N} = 1$ SQCD, we have only the x^7 direction (FI D -term) for the transport to maintain the supersymmetries of the vacua. The worldvolumes for the branes are summarized in table 3.

After turning on the FI D -term parameter (parallel-transporting the NS'5-brane in the x^7 direction), k D2-branes can stretch between the NS'5-brane and the N_c D4-branes, see figure 5(a). This is a description of the electric side of the SQCD. In the magnetic side, we put the D2-branes stretched between the NS5-brane and the $(N_f - N_c)$ D4-branes, see figure 5(b).¹⁰

Non-BPS solitonic strings in magnetic theory of massive SQCD. Let us consider the possibility of the existence of the solitonic strings in the magnetic theory of the *massive*

¹⁰Field theoretical properties of the semilocal strings and relations to the brane configurations and to the Seiberg dualities are studied in [19].

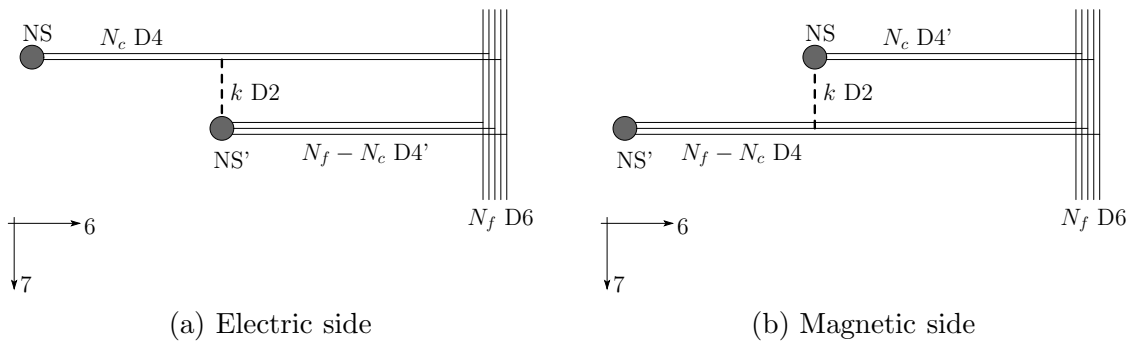


Figure 5: Brane configurations for the 1/2 BPS semilocal strings in $\mathcal{N} = 1$ SQCD.

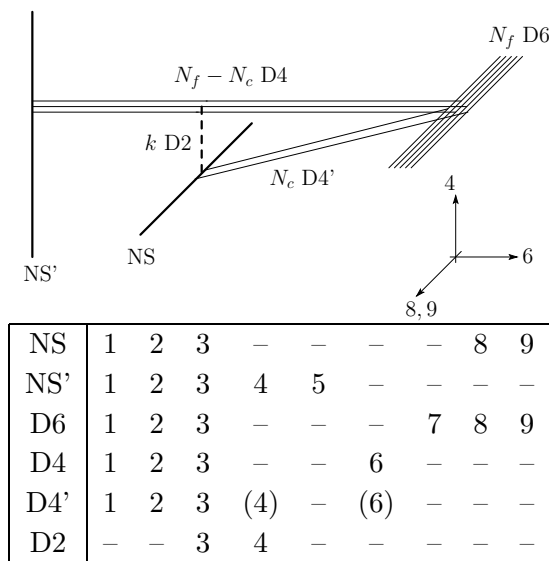


Figure 6: Brane realization (D2-branes) of the solitonic strings in the meta-stable vacua.

$\mathcal{N} = 1$ SQCD without the FI parameter, explained in section 2.1. The 1/2 BPS semilocal strings in the $\mathcal{N} = 2$ model required a non-vanishing FI term for them to exist. Instead, as has been described, the SQCD has a non-zero “quark mass” μ^2 , which behaves in the magnetic side quite similarly to the FI term. This “mass” term in fact supports the solitonic string in the dual SQCD, as we will see in the following.

We can easily find a brane configuration for the solitonic string, by applying the idea of [11]. In the brane realization of the magnetic theory provided by [3–5], we put a D2-brane suspended between the NS5-brane and the $(N_f - N_c)$ D4-branes. See figure 6. Because the D2-brane tends to minimize its length, it is perpendicular to both the NS5-brane and the D4-branes.

From the brane configuration, we can extract several properties of the solitonic strings. All of those are found to be consistent with our field theory analyses which will be presented in the next section.

- **Tension of the string.** The length of the D2-brane measures the tension. It is found

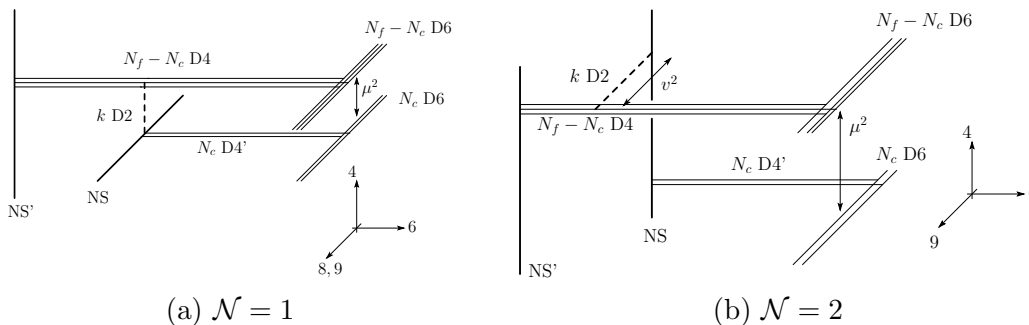


Figure 7: Non-BPS solitonic strings in massive dual $\mathcal{N} = 1$ SQCD

to be proportional to μ^2 , the quark mass in the original SQCD, since the length is identical to the distance between the D6-branes and the NS5-brane in the x^4 axis.

- Supersymmetries.** The meta-stable vacua completely break supersymmetries by themselves. This has been realized as the tilted N_c D4'-branes in the brane configuration [3–5]. We have to stress that the semilocal strings should also break the supersymmetries completely, since the N_f D6-branes along the directions 0123789 and the D2-brane along 034 are incompatible with supersymmetries. So the solitonic string in the supersymmetry-breaking meta-stable vacua are non-BPS (\equiv supersymmetry-breaking)¹¹ semilocal strings.
- Existence of non-BPS strings in supersymmetry-preserving vacua.** Before proceeding to the field theory analysis, we study brane configurations of non-BPS solitonic strings, not in the supersymmetry-breaking vacua, but in the supersymmetry-preserving vacua, in the magnetic theory of the $\mathcal{N} = 1$ massive SQCD. As reviewed briefly at the end of section 2.1, if we replace the “mass” term $\mu^2 \text{Tr} M$ in the superpotential (2.2) by $\text{Tr}[mM]$ where $m = \text{diag}(\mu^2, \dots, \mu^2, 0, \dots, 0)$, the resultant vacuum becomes supersymmetric. The solitonic strings are again realized D2-branes between $N_f - N_c$ D4-branes and NS5-brane. It appears that the brane configuration preserves some supersymmetries, but note that the D2-branes and the D6-branes are still in conflict with any compatible supersymmetries. Thus the solitonic string in this supersymmetric vacuum is again non-BPS.

The solitonic string in this vacuum is not a semilocal string but a non-Abelian string with the orientational moduli CP^{N-1} , because there is no global symmetry $SU(N_f)$ from the first place. The brane configuration in figure 7(a) is quite similar to that in figure 4. However, properties of the solitonic strings, namely D2-branes suspended D4-branes and NS5-brane, are not so similar: The solitonic strings in figure 4 are 1/2 BPS semilocal strings which have orientational moduli and size moduli, while those in figure 7(a) are non-BPS non-Abelian strings which doesn't have the size moduli.

¹¹Note that in our terminology, “non-BPS” means supersymmetry-breaking, and not the saturation of the Bogomol’nyi bound.

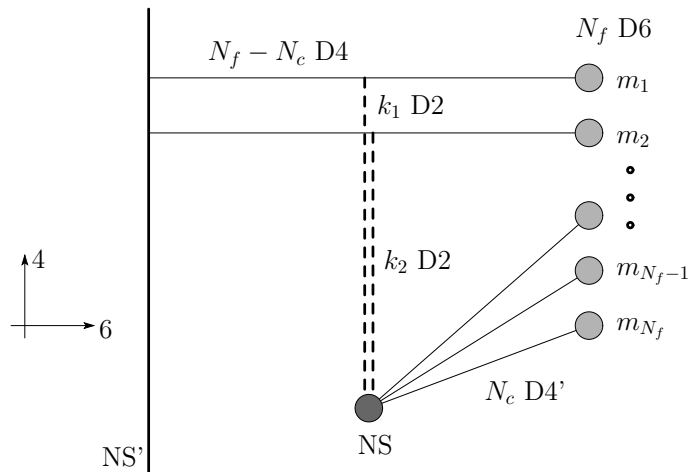


Figure 8: Brane configuration of various strings with different tensions.

To illustrate this distinction, we draw an $\mathcal{N} = 2$ D-brane configuration figure 7(b) which is much more similar to the brane configuration figure 7(a). This figure 7(b) shows a massive Hanany-Tong setup in which the vacuum is not degenerate, since the non-degenerate masses break $SU(N_f)$ down to $SU(N)$ explicitly. So the solitonic string in figure 7(b) is not a 1/2 BPS semilocal non-Abelian string but a 1/2 BPS non-Abelian string without the size moduli.

- **Existence of multi-tension strings.** In general, we can put non-degenerate “masses” $m = \text{diag}(m_1, m_2, \dots, m_{N_f})$. The corresponding D-brane configuration is shown in figure 8. Stable vacua are those in which $N_f - N_c$ horizontal D4-branes connect the NS5-brane and the $N_f - N_c$ D6-branes associated with the $N_f - N_c$ large masses $\{m_1, \dots, m_{N_f - N_c}\}$. The remaining N_c D4'-branes are suspended between the NS5-brane and the N_c D6-branes associated with the light masses $\{m_{N_f - N_c + 1}, \dots, m_{N_f - 1}, m_{N_f}\}$. Obviously one can put the D2-branes between the horizontal D4-branes and the NS5-brane. There are $N_f - N_c$ kinds of D2-branes, depending on which D4-brane (labeled by $i = 1, \dots, N_f - N_c$) the D2-brane ends on. The tension of the solitonic string comes from the distance between the i -th D4-brane and the NS5-brane, hence is proportional to m_i .

In the next section, we will give explicit solutions of the solitonic strings in the magnetic theory of the SQCD, and show that indeed these properties are equipped with the strings.

4. Vortex string solutions in meta-stable vacua

In this section, we explicitly construct the solitonic string solutions of the equations of motion in the magnetic theory of the massive SQCD. We find that the resultant strings have the properties expected from the brane configurations: the tensions, the supersymmetries, and the various species of the strings.

4.1 Classical solutions

The existence of the solution to the equations of motion can be seen by considering the following field configurations

$$M = 0, \quad \frac{1}{\sqrt{2}}(q + \tilde{q}^\dagger) \equiv \phi, \quad \frac{1}{\sqrt{2}}(q - \tilde{q}^\dagger) \equiv \tilde{\phi} = 0. \quad (4.1)$$

Interestingly, this assumption leads to a system which is almost identical to that of the semilocal strings. The only difference is the vacuum energy, as we will see shortly below: in our case the vacuum energy is non-vanishing and thus all the supersymmetries are always broken, while in the usual case of the BPS semilocal strings some supersymmetries are preserved at the vacuum.

With the truncation (4.1), the Lagrangian takes the form¹²

$$\tilde{\mathcal{L}} = \text{Tr}_c \left[-\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} - \mathcal{D}_\mu \phi \mathcal{D}^\mu \phi^\dagger \right] - \frac{|h^2|}{4} \text{Tr}_f \left[\left(\phi^\dagger \phi - 2\mu^2 \mathbf{1}_{N_f} \right)^2 \right], \quad (4.3)$$

where ϕ is a complex matrix-valued field whose size is N by N_f . Here we gauged the $U(1)_B$ symmetry and unify it with the $SU(N)$ so that the full gauge symmetry becomes $U(N)$, i.e. we put the gauge coupling of the $U(1)_B$ to be equal to that of the $SU(N)$. Note that the $U(1)_B$ which we gauged corresponds to the original gauged baryonic $U(1)$ symmetry in the electric side and we took Seiberg's duality only for $SU(N_c)$. Unifying $U(1)_B$ gauge coupling and $SU(N)$ gauge coupling here is just for a convenience. We will consider the generic case with non-coincident gauge couplings in the appendix. Note that the above potential term can be written equivalently as

$$\tilde{V} = |h^2 \mu^4| (N_f - N) + \frac{|h^2|}{4} \text{Tr}_c \left[\left(\phi \phi^\dagger - 2\mu^2 \mathbf{1}_N \right)^2 \right]. \quad (4.4)$$

As we will show shortly, the system is identical to that of the non-Abelian semilocal strings plus the additive cosmological constant, the first term in (4.4). Because of this cosmological constant, all the supersymmetries are always broken, although the system is similar to that of the semilocal strings. The equation of motion of the truncated model is

$$\mathcal{D}_\mu \mathcal{D}^\mu \phi = \frac{\partial \tilde{V}}{\partial \phi^\dagger}, \quad \frac{1}{g^2} \mathcal{D}^\mu F_{\mu\nu} = -\frac{i}{2} \left(\phi \mathcal{D}_\nu \phi^\dagger - \mathcal{D}_\nu \phi \phi^\dagger \right), \quad (4.5)$$

where $\mu, \nu = 1, 2$ for the vortex strings extending along the x^3 axis. Notice that the cosmological constant in (4.4) does not appear in the equation of motion. The equation of motion for $\tilde{\phi}$ is satisfied with $\tilde{\phi} = 0$.

This equation of motion (4.5) reduces to that for the well-known Abelian semilocal strings when we choose $N_f > N = 1$, and their solutions were studied in detail [13]. We

¹²When we take $g^2 \rightarrow \infty$ and simultaneously $|h| \rightarrow \infty$, this model becomes a non-linear sigma model whose target space is

$$\mathcal{M}_{\text{target}} = \left\{ \phi^\dagger \phi - 2\mu^2 \mathbf{1}_N \right\} / [SU(N) \times U(1)_B] \simeq Gr_{N_f, N}. \quad (4.2)$$

Note that in this limit we discard the infinite cosmological constant $(N_f - N)|h^2 \mu^4| \rightarrow \infty$.

can find a minimal string solution of the non-Abelian semilocal string ($N_f > N \geq 2$) for the equation (4.5) by embedding the U(1) semilocal string solution in the model of $N_f - N + 1$ flavors as

$$F_{12} = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & F_{12}^* \end{pmatrix}, \quad \phi = \begin{pmatrix} \sqrt{2}\mu & & & \\ & \ddots & & \\ & & \sqrt{2}\mu & \\ & & & \phi_0^* \ \phi_1^* \ \cdots \ \phi_{N_f-N}^* \end{pmatrix}, \quad (4.6)$$

where we denoted F_{12}^* and $\{\phi_a^* (a = 0, 1, \dots, N_f - N)\}$ as solutions of the minimal winding U(1) semilocal string. This is the solution of the solitonic strings in the supersymmetry-breaking meta-stable vacua. As we mentioned in section 2.3, the vacuum has $SU(N)_{c+f}$ color-flavor locking symmetry. This symmetry is broken down to $U(1) \times SU(N - 1)$ by the single non-Abelian semilocal string (4.6), so that the solution has internal orientational zero modes (Nambu-Goldstone modes) $CP^{N-1} \simeq SU(N)/[SU(N - 1) \times U(1)]$.

When embedding the U(1) semilocal solution to the non-Abelian gauge theory, we have to choose a single U(1) gauge sub-sector in the U(N). This corresponds precisely to choosing a single D4-brane (among $N = N_f - N_c$ of them) on which the D2-brane should end. The tension of the non-Abelian semilocal string is proportional to μ^2 . This is again consistent with the D-brane picture, where the length of the D2-brane (proportional to μ^2) is determined only by the distance between the NS5-brane and the $(N_f - N_c)$ D4-branes and independent of the other parameters.

Now we consider a special case where the Higgs self-coupling h satisfies

$$g^2 = |h|^2, \quad (4.7)$$

although this would not be always satisfied for the dual of SQCD. In this case the non-Abelian semilocal string saturates the Bogomol'nyi bound of the theory with (4.4) (the tension is given by $4\pi\mu^2$), though the supersymmetries are broken because of the cosmological constant. In this case the second order differential equation (4.5) reduces to the 1st order differential equation (3.6) with replacing v^2 with μ^2 . It is known that the minimal BPS semilocal vortex has the size moduli in addition to the orientational moduli CP^{N-1} . Furthermore, any repulsive or attractive force does not appear between separated strings due to the saturation, like the ordinary 1/2 BPS solitons. As a result the moduli space of the non-Abelian semilocal string in the meta-stable SUSY breaking vacua is completely the same as that for the 1/2 BPS non-Abelian semilocal string in $\mathcal{N} = 2$ supersymmetric vacua [11, 16, 17, 20].

We would like to stress that our non-Abelian semilocal string solutions are always non-BPS (breaking all the supersymmetries) even for the case where the energy bound is saturated ($g^2 = |h|^2$). One can easily show that the 1/2 BPS equation (3.6) is inconsistent with the $\mathcal{N} = 1$ super-transformation of the gaugino $\delta_\epsilon \lambda = \sigma^{\mu\nu} \epsilon F_{\mu\nu} + i\epsilon D$ with $D \sim qq^\dagger - \tilde{q}^\dagger \tilde{q} = 0$ in the selected sector (4.1). It is somehow surprising that the supersymmetry-breaking semilocal string saturates the Bogomol'nyi energy bound when $g^2 = |h|^2$ and the

non-BPS semilocal strings behaves as if they were 1/2 BPS semilocal strings.¹³ Actually they have free moduli parameters of positions, internal orientations and sizes. From the D-branes viewpoint this is not intuitive at all since the angle between the D2-branes and D4'-branes is not the right angle.

Due to the saturation in the case where $g^2 = |h|^2$, the string is stable in the selected sector (4.1). Later we will study the fluctuation orthogonal to the truncation (4.1), and find that the string is stable against all the fluctuations when (4.7) is satisfied.

4.2 Stability of solitonic string

Let us study the stability of the ANO string in the magnetic theory of the massive SQCD, which we found in the previous subsection. We will find that the vortex is stable for

$$g^2 \geq |h|^2. \tag{4.8}$$

It is known [13] that Abelian semilocal vortices are unstable for $g^2 < |h|^2$. In this parameter region the strings can reduce their tension by fattening themselves (increasing their widths). Therefore, any possible region of the parameter space for the ANO string to be stable should be within (4.8). In our truncated theory with (4.1), this Abelian situation corresponds to $N = 1$. For our general non-Abelian case, we can show that the non-Abelian semilocal string is stable again for (4.8), as in the following. The solution (4.6) for $g^2 > |h|^2$ has vanishing $\{\phi_1^*, \dots, \phi_{N_f-N}^*\}$, thus we consider a fluctuation from this ANO solution. Matrix elements of the scalar fluctuation are labeled as

$$\phi = \begin{pmatrix} \sqrt{2}\mu \mathbf{1}_{N-1} + \delta\phi_I & \delta\phi_{II} & \delta\phi_{III} \\ \delta\phi_{IV} & \phi_0^* + \delta\phi_V & \delta\phi_{VI} \end{pmatrix} \tag{4.9}$$

Substituting this and a similar decomposition of the gauge fields into the Lagrangian, it is easy to show that the ‘‘semilocal sectors’’ $\delta\phi_{III}$ and $\delta\phi_{VI}$ decouple from the others (which are fluctuations of a non-Abelian local string with $N = N_f$). This latter fluctuations are expected to be stable for any g and h . $\delta\phi_V$ is found to be identical to the fluctuation of the Abelian local vortex (ANO), thus is stable. The sector $\delta\phi_I$ couples to the vacuum expectation value $\sqrt{2}\mu$ and is Higgsed to be massive and stable. A combination of $\delta\phi_{II}$ and $\delta\phi_{IV}$ produces the orientational moduli which are normalizable, localized and massless modes. The other combinations are massive and stable, through a Higgs mechanism, which would be understood in a unitary gauge, for example. Our main concern is in the semilocal sector. We find that $\delta\phi_{III}$ does not have a potential term and thus is a massless bulk mode which is non-normalizable. This is a Nambu-Goldstone mode associated with the global symmetry breaking by the vacuum. $\delta\phi_{VI}$ has the same expression as the fluctuation of the Abelian semilocal string, therefore we know that for $g^2 > |h|^2$ it is massive and stable, as

¹³Our non-BPS string is not a kind of the F -term vortex string in $\mathcal{N} = 1$ theory [21, 22] which also behaves as if it was a 1/2 BPS string though it doesn't preserve the $\mathcal{N} = 1$ supersymmetry. The F -term string preserves 1/2 supersymmetry when it is embedded into an appropriate $\mathcal{N} = 2$ theory. However, our non-BPS strings cannot be simply embedded into any $\mathcal{N} = 2$ model, partly because the scalar field M is not an adjoint scalar field of the $U(N)$. This may be also obvious from the viewpoint of the D-brane configuration, see figure 6.

mentioned above. For $g^2 = |h|^2$, this $\delta\phi_{\text{VI}}$ gives the massless moduli $\{\delta\phi_1^*, \dots, \delta\phi_{N_f-N}^*\}$ as indicated in (4.6). In sum, the solution (4.6) is stable for (4.8) in the truncated model (4.1).

Second, we consider fluctuations orthogonal to the truncation (4.1). Fluctuation of the field M does not provide any instability, because in the potential term (2.4) the field M appears in a positive-semi-definite form. Thus we need to study the fluctuation of q and \tilde{q} . We redefine the fields as

$$\phi \equiv \frac{1}{\sqrt{2}}(q + \tilde{q}^\dagger), \quad \tilde{\phi} \equiv \frac{1}{\sqrt{2}}(q - \tilde{q}^\dagger). \quad (4.10)$$

(Previously we have put $\tilde{\phi} = 0$.) The $\tilde{\phi}$ potential can be written, up to its quadratic order, as

$$\text{Tr}_f \left[\mu^2 h^2 |\tilde{\phi}^\dagger \tilde{\phi} + \frac{|h|^2}{2} \phi^\dagger \tilde{\phi} \tilde{\phi}^\dagger \phi \right] + \frac{1}{4}(g^2 - |h|^2) \text{Tr}_c \left[(\tilde{\phi} \phi^\dagger + \phi \tilde{\phi}^\dagger)^2 \right]. \quad (4.11)$$

The term linear in $\tilde{\phi}$ vanishes identically, and consequently there is no mixing between $\tilde{\phi}$ and the fluctuation of ϕ . Since (4.11) is a sum of perfect squares, the fluctuation is stable when (4.8) is satisfied. Therefore we conclude that the ANO string is stable for $g^2 \geq |h|^2$.

Irrespective of the values of the couplings, there exist massless fluctuations. The eigenfunction of the fluctuation is given for example by

$$\delta\phi_{(i,j)} = \phi^{\text{ANO}}(x^1, x^2) f_{(i,j)}(x^0, x^3) \quad (4.12)$$

where (i, j) labels the matrix elements, x^3 is the direction along the embedded ANO string, and ϕ^{ANO} is the ANO vortex solution in the Abelian Higgs model. This is certainly expected, since this is a Nambu-Goldstone mode associated with the spontaneous breaking of the global symmetry $\text{SU}(N_f)$. If we rotate the embedded ANO solution a little bit by the global symmetry $\text{SU}(N_f)$, then we obtain this massless fluctuation.

4.3 Multi-tension non-BPS vortex strings

Instead of choosing all the “mass” equal to each other (to be equal to μ^2), we may choose a superpotential $W = -h \text{Tr}_f [mM]$, as described in section 2.1. We align the mass eigenvalues as $m = \text{diag}(m_1, \dots, m_{N_f})$ with $m_1 \geq \dots \geq m_{N_f} \geq 0$ so that the vacuum is stable locally (see [3] for discussions on the vacuum stability from the brane configurations).

Obviously we find the embedding of the ANO string as before,

$$\phi_{(1,1)} = \phi^{\text{ANO}}(x^1, x^2), \quad A_{(1,1)}^\mu = (A^{\text{ANO}})^\mu(x^1, x^2), \quad \text{the others} = 0. \quad (4.13)$$

Here the subscript means again the (i, j) component. For $g^2 = |h|^2$, the tension of this ANO string is proportional to m_1 , as predicted from the brane configurations.

One can embed the string in one of the other sectors, by replacing the above $(1, 1)$ by (j, j) for $j = 2, \dots, N_f - N_c$. This is again obviously a solution to the equations of motion. The ANO string has its tension $\propto m_i$, which is consistent with the brane configurations. Thus we actually obtain multi-tension vortices.

The analysis of the stability of the string is almost similar to the case of the semilocal strings. For example, if we embed an ANO string in the (1, 1) component as in (4.13), we find the fluctuation Lagrangian for $\phi_{(j,1)}$ for $j = 2, \dots, N_f$ as

$$-|\mathcal{D}_\mu^{\text{ANO}} \phi_{(j,1)}|^2 - \frac{|h|^2}{2} (|\phi^{\text{ANO}}|^2 - 2m_j) |\phi_{(j,1)}|^2. \quad (4.14)$$

For the semilocal case $m_1 = \dots = m_{N_f} = \mu^2$, we know that this fluctuation is stable for $g^2 \geq |h|^2$. In the present case, the above fluctuation Lagrangian is obtained just by replacing μ^2 by m_j . Since m_j is smaller than m_1 , the tachyonic instability is now found to be improved, and the situation is better for the stability. Thus for $g^2 \geq |h|^2$ the ANO solution is stable against this kind of fluctuations. For the fluctuations orthogonal to the truncation (4.1), a similar computation shows that the potential is the same as (4.11) except that we replace $|\mu|^2$ in the first term by the matrix m . The argument for the stability is the same, and we find that the string is stable for $g^2 \geq |h|^2$ against $\tilde{\phi}$. Therefore, in summary, we find that the ANO string embedded in (1, 1) sector is stable for $g^2 \geq |h|^2$.

When $m_{N+1} = \dots = m_{N_f} = 0$, the vacuum admits supersymmetries, and the cosmological constant vanishes. In the corresponding brane configuration, all the D4-branes become parallel, which is a manifestation of the supersymmetry restoration. Let us consider the embedding of the ANO string as before, in this supersymmetric vacuum. The embedded solitonic string breaks the supersymmetries even for $g^2 = |h|^2$, similarly to the case of the non-Abelian semilocal strings dealt with at the end of section 4.1. This is consistent with what we have found in the brane configurations.

5. Conclusion and discussions

The vortex strings which we have found in this paper can be applied to various situations, such as phenomenological model building and cosmologies. Since the meta-stable vacua found in [2] provide us with a new path to break supersymmetries at low energy, the possible existence of solitons in the vacua may affect any story on the vacua. We have found that the $U(N_c)$ and the $SO(N)$ SQCD have vortex strings. The vortex strings in the $U(N_c)$ (and the $SO(N_c)$ with $N_f = N_c - 2$) are similar to the semilocal strings. The vortex strings in general $SO(N_c)$ are \mathbf{Z}_2 strings. The existence and the properties of the $U(N_c)$ strings have been found from brane configurations. This time again, stringy technique turned out to be quite useful in finding field theoretical solutions and their properties.

Several discussions and comments are in order, which we hope to get back to in the future.

- Implication to cosmologies. The vortex strings found in this paper can be thought of as cosmic strings. One can argue that when the universe is cooling down, the energy scale gets smaller than the typical scale determined by the Seiberg duality, and if eventually the supersymmetry-breaking meta-stable vacua are chosen somehow, the vortex strings may form then. It is noteworthy that, even though the vacua is non-supersymmetric and consequently the cosmic string is non-BPS, when the

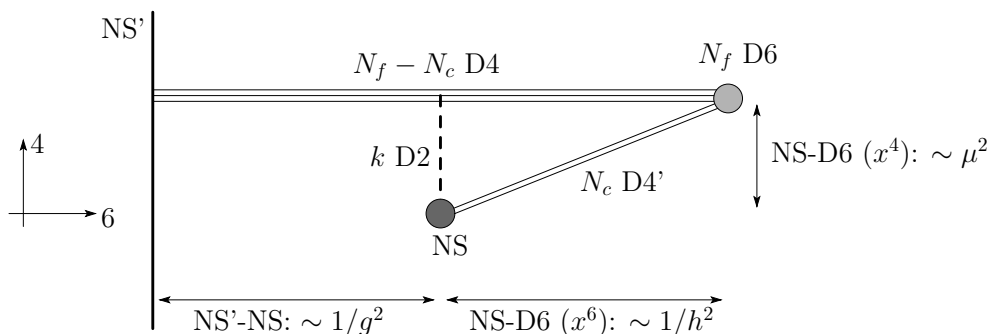


Figure 9: The relation between the field theory couplings and locations of the branes.

gauge coupling g is equal to the scalar self coupling $|h|$, the solution itself is the same as the BPS vortex strings. Therefore one can use various results on the BPS vortex strings found in particular in the moduli matrix formalism [20] (see [23]). For example, reconnection probability of cosmic strings is important for evaluating number density of the cosmic strings and consequently possible observation of them. The cosmic strings found in this paper with $g = |h|$ provides an analytic study of the reconnection of cosmic strings in a non-supersymmetric background, which is quite interesting.

- Renormalization group flow and stability of the strings. The stability analysis of section 4 has been done at the tree level of the magnetic theory. There we have found that for $g \geq |h|$ the string is stable. To argue the stability in more realistic situations, one needs to consider quantum effects, including the renormalization group flow of the couplings. It is important to know if the stability condition $g \geq |h|$ is satisfied or not at the energy scale where the classical soliton solutions are reliable. The stability depends on the beta functions and the precise values of the coupling constants at some energy scale. However, it is difficult to determine the Yukawa coupling h precisely, though we know it is $\mathcal{O}(1)$ at the typical energy scale appearing in the Seiberg duality.
- Prediction of the stability from brane configurations. As studied in section 4, the vortex string is classically stable for $g \geq |h|$. It is interesting if this condition can be seen in the brane configurations. The couplings g and h can be interpreted [5] as the distances between D-branes and NS5(NS'5)-branes (at least for the massless SQCD), as shown in figure 9. The distance between NS5-NS'5 along the x^6 axis corresponds to the gauge coupling as $\sim 1/g^2$, that between NS5-D6 along the x^6 axis to $\sim 1/|h|^2$ and NS5-D6 along the x^4 axis to the “mass” $\sim \mu^2$. When the distance between NS5-NS'5 is smaller than the distance NS5-D6 along the x^6 axis, we have $g^2 > |h|^2$, so the field theory results shows that the ANO string (the semilocal string with vanishing width) stably exists there. When the distance between NS5-NS'5 is smaller than the distance NS5-D6 along the x^6 axis, we have $g^2 < |h|^2$ so any string

solution does not exist in field theory. The question is if one can read these stability information purely in the brane configuration. Our tentative conclusion here is that we can read the tendency of the instability, but the actual relation $g \geq |h|$ is difficult to be seen, as is explained below.

The moduli space of the solitonic string should be seen along the argument given by Hanany and Tong [11], as a low energy field theory on the D2-brane. As seen in section 3.2, for the semilocal strings in the $\mathcal{N} = 2$ gauge theory, massless excitations on the D2-brane came from strings in the following three sectors: (i) Z from D2-D2, (ii) ψ from D2-D4 and (iii) $\tilde{\psi}$ from D4'-D2 in figure 4. (i) is relevant for the transverse location of the string, (ii) is for the orientational moduli, and (iii) is for the width of the string. Apparently the instability of the semilocal string should be related to (iii).

Suppose that NS5-brane is closer to the D6-branes, compared to the distance to the NS'5-brane. As can be found in figure 6 or in figure 9, the D2-D4' string is naively expected to become tachyonic, since the angle between the D2-brane and the D4'-brane is not the right angle. Consequently, the solitonic string is expected to be unstable against the change of the width. This is consistent with the field theory analysis for $g < |h|$. When one moves the NS5-brane toward the NS'5-brane in the x^6 direction, the angle becomes closer to the right angle, thus the tachyonic instability is improved. This is again consistent with the field theory analysis, since $g/|h|$ is getting larger.

However, a strange discrepancy between the D-brane picture and the field theory analysis appears when one continues to make the NS5-brane approach the NS'5-brane. The angle is still less than the right angle and thus the D2-D4' string seems to be tachyonic, while in the field theory analysis the vortex becomes stable. This tells us that the angle is not directly corresponding to the mass of the moduli parameter, somehow, although in the $\mathcal{N} = 2$ case [11] this has been assumed even in the presence of the NS5-brane and it worked. Our situation breaks supersymmetries, that might be a reason why it does not work now.

A similar argument can apply for the case of multi-tension strings found in section 4.3. We have found in (4.14) that the string is more stable for smaller m_j . This phenomenon is compatible with the brane configuration. Now the D2-brane is ending on the D4-brane whose position in the x^4 - x^5 plane is given by the complex parameter m_1 . When other mass parameters $|m_i|$ ($i = N + 1, \dots, N_f$) are smaller than this $|m_1|$, the angle between the D2-brane and one of the D4-branes labeled by $i = N + 1, \dots, N_f$ is closer to the right angle, thus the instability of the D2-brane caused by the tachyonic strings connecting the D2-brane and the D4-branes is smaller. However, again the problem of vanishing instability still remains in the brane story.

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A. ANO like vortex in the $SU(N) \times U(1)_B$ gauge theory

In this appendix, we construct ANO like vortex in the magnetic theory where the $SU(N)$ gauge coupling g and the $U(1)_B$ gauge coupling e are different. Here we consider the field configuration (4.1) only and then the equations of motion is

$$\mathcal{D}_\mu \mathcal{D}^\mu \phi = \frac{\partial \tilde{V}}{\partial \phi^\dagger}, \quad (\text{A.1})$$

$$\frac{1}{g^2} [\mathcal{D}^\mu F_{\mu\nu}]^{\text{SU}(N)} = -\frac{i}{2} \left[(\phi \mathcal{D}_\nu \phi^\dagger - \mathcal{D}_\nu \phi \phi^\dagger) \right]^{\text{SU}(N)}, \quad (\text{A.2})$$

$$\frac{1}{e^2} [\mathcal{D}^\mu F_{\mu\nu}]^{\text{U}(1)} = -\frac{i}{2} \left[(\phi \mathcal{D}_\nu \phi^\dagger - \mathcal{D}_\nu \phi \phi^\dagger) \right]^{\text{U}(1)}, \quad (\text{A.3})$$

where $[\dots]^{\text{SU}(N)}$ and $[\dots]^{\text{U}(1)}$ means the traceless part and the trace part of the $N \times N$ matrix, respectively. It is easy to see that the solution of the equations of motion in this restricted configuration space is also the solution in the full configuration space. Here we take the following axial ansatz for the solution (see [16, 24]):

$$A_2 - iA_1 = \frac{n}{r} \begin{pmatrix} b(r) & & & \\ & \ddots & & \\ & & b(r) & \\ & & & a(r) \end{pmatrix} e^{i\theta}, \quad \phi = \begin{pmatrix} v(r) & & & \\ & \ddots & & \\ & & v(r) & \\ & & & u(r)e^{in\theta} \ 0 \ \dots \ 0 \end{pmatrix}, \quad (\text{A.4})$$

where n is an integer and all the other components of A_μ are zero. Then the equations of motion become

$$\begin{aligned} u'' + \frac{u'}{r} - n^2 \frac{(1-a)^2}{r^2} u &= \frac{|h|^2}{2} (u^2 - 2\mu^2) u, \\ v'' + \frac{v'}{r} - n^2 \frac{b^2}{r^2} v &= \frac{|h|^2}{2} (v^2 - 2\mu^2) v, \\ \left(\frac{N-1}{g^2} + \frac{1}{e^2} \right) \left(a'' - \frac{a'}{r} \right) + \left(\frac{1}{e^2} - \frac{1}{g^2} \right) (N-1) \left(b'' - \frac{b'}{r} \right) &= -N(1-a)u^2, \\ \left(\frac{1}{g^2} + \frac{N-1}{e^2} \right) \left(b'' - \frac{b'}{r} \right) + \left(\frac{1}{e^2} - \frac{1}{g^2} \right) \left(a'' - \frac{a'}{r} \right) &= Nbv^2, \end{aligned} \quad (\text{A.5})$$

where $a' = \frac{d}{dr}a(r)$ and so on. The boundary conditions at the origin should be

$$a(0) = b(0) = u(0) = v'(0) = 0 \tag{A.6}$$

because $\phi, \partial_\mu\phi$ and $F_{\mu\nu}$ are regular at the origin. Actually,

$$a(r) = \mathcal{O}(r^2), b(r) = \mathcal{O}(r^2), u(r) = \mathcal{O}(r^n), v'(r) = \mathcal{O}(r^1), \tag{A.7}$$

are consistent with (A.5). The boundary conditions at $r = \infty$ should be

$$a(\infty) = 1, b(\infty) = 0, u(\infty) = v(\infty) = \sqrt{2}\mu, \tag{A.8}$$

which is also consistent with (A.5). Then, with these eight consistent boundary conditions we can solve (A.5) in principle as in the case of the simple ANO vortex though we do not carry out it here. Therefore we expect (A.4) is the vortex solution with n charge. Note that

$$\pi_1 \left(\frac{\text{SU}(N) \times \text{U}(1)_B}{\mathbf{Z}_N} \right) = \mathbf{Z} \tag{A.9}$$

and the vortex with the minimal charge winds the $\text{U}(1)_B$ $1/N$ times.

Finally, we comment on the stability of the solutions. If $|e| > |h|$ these solutions is expected to be stable as in the case $e = g$. A more complete analysis of the stability is desired, but, we will leave it as a future problem.

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